Formale Sprachen und Automaten

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Lösung zu Aufgabe 6.2:

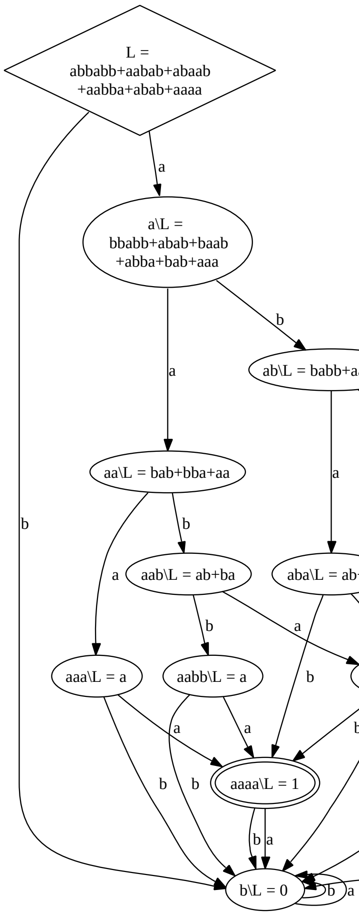
(i) Wir berechnen die Wortmengen $w\backslash L := \{ v \in \Sigma^* \mid wv \in L \}$ für die verschiedenen Wörter $w \in \Sigma^*$, bis keine neuen Mengen mehr auftreten:

$$
\begin{array}{rcl}
\epsilon\backslash L &=& \{abbabb, aabab, abaab,\\
&& \;aabba, abab, aaaa\}\\
a\backslash L &=& \{bbabb, abab, baab, abba, bab, aaa\}\\
b\backslash L &=& \emptyset,\\
aa\backslash L &=& \{bab, bba, aa\}\\
ab\backslash L &=& \{babb, aab, ab\}\\
aaa\backslash L &=& \{a\}\\
aab\backslash L &=& \{ab, ba\}\\
aba\backslash L &=& \{ab, b\}\\
abb\backslash L &=& \{abb\}
\end{array}
\qquad
\begin{array}{rcl}
aaaa\backslash L &=& \{\epsilon\}\\
aaab\backslash L &=& \emptyset\\
aaba\backslash L &=& \{b\}\\
aabb\backslash L &=& \{a\}\\
abaa\backslash L &=& \{b\}\\
abab\backslash L &=& \{\epsilon\}\\
abba\backslash L &=& \{bb\}\\
abbb\backslash L &=& \emptyset
\end{array}
\qquad
\begin{array}{rcl}
aaaaa\backslash L &=& \emptyset\\
aaaab\backslash L &=& \emptyset\\
aabaa\backslash L &=& \emptyset\\
aabab\backslash L &=& \{\epsilon\}\\
aabba\backslash L &=& \{\epsilon\}\\
aabbb\backslash L &=& \emptyset\\
abaaa\backslash L &=& \emptyset\\
abaab\backslash L &=& \{\epsilon\}\\
ababa\backslash L &=& \emptyset\\
ababb\backslash L &=& \emptyset\\
abbaa\backslash L &=& \emptyset\\
abbab\backslash L &=& \{b\}
\end{array}
$$

und schließlich

$$
\begin{array}{rcl}
aababa\backslash L &=& \emptyset\\
aababb\backslash L &=& \emptyset\\
aabbaa\backslash L &=& \emptyset\\
aabbab\backslash L &=& \emptyset
\end{array}
\qquad
\begin{array}{rcl}
abaaba\backslash L &=& \emptyset\\
abaabb\backslash L &=& \emptyset\\
abbaba\backslash L &=& \emptyset\\
abbabb\backslash L &=& \{\epsilon\}
\end{array}
\qquad
\begin{array}{rcl}
abbabba\backslash L &=& \emptyset\\
abbabbb\backslash L &=& \emptyset
\end{array}
$$

Ist $w\backslash L = \emptyset$, so sind auch $wa\backslash L = \emptyset = wb\backslash L$, und folglich alle $wu\backslash = \emptyset$. Statt des Baums (ohne die Quotienten der leeren Mengen) geben wir gleich den (deterministischen) Automaten $\mathcal{A} = (Q, \xrightarrow{a}, \xrightarrow{b}, I, F)$ mit $Q = \{ w\backslash L \mid w \in \Sigma^* \}$, $\xrightarrow{a} = \{ (w\backslash L, wa\backslash L) \mid w \in \Sigma^* \}$, $\xrightarrow{b} = \{ (w\backslash L, wb\backslash L) \mid w \in \Sigma^* \}$, $I = \{\epsilon\backslash L\}$, $F = \{ w\backslash L \mid w \in L \}$ an:



(ii) Um den Automaten der Sprachen $w\backslash L$ für $L = L((a+b)(a+b)^*a+a(bb)^*a^*)$ zu bestimmen, schreiben wir statt $w\backslash L(r)$ einfach $w\backslash r$ mit regulären Ausdrücken r. Man beachte, daß $w\backslash(r+s) = w\backslash r + w\backslash s$ und wegen $r^* = rr^* + 1$ insbesondere

$$\begin{aligned}
a\backslash(a+b)^* &= a\backslash((a+b)(a+b)^* + 1) \\
&= a\backslash(a(a+b)^*) + a\backslash(b(a+b)^*) + a\backslash 1 \\
&= (a+b)^* + 0 + 0 \\
&= (a+b)^*
\end{aligned}$$

gilt, und analog $b\backslash(a+b)^* = (a+b)^*$ und

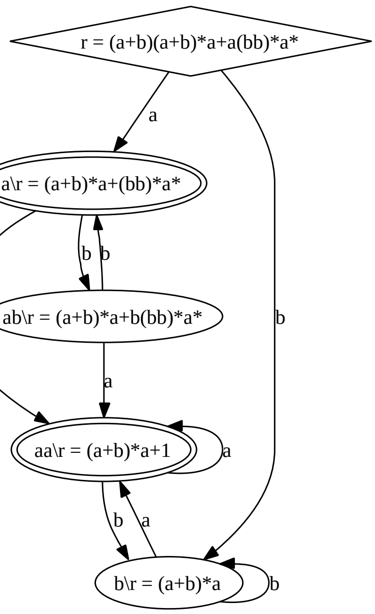
$$a\backslash((a+b)^*r) = a\backslash(a(a+b)^*r + b(a+b)^*r + r) = (a+b)^*r + a\backslash r.$$

Daher erhalten wir

$$\begin{aligned}
r &:= (a+b)(a+b)^*a + a(bb)^*a^* \\
a\backslash r &= (a+b)^*a + (bb)^*a^* \\
b\backslash r &= (a+b)^*a \\
aa\backslash r &= a\backslash(a\backslash r) = a\backslash((a+b)^*a) + a\backslash((bb)^*a^*)
\end{aligned}$$

$$
\begin{array}{rcl}
 & = & (a+b)^*a + a\backslash a + a\backslash(bb(bb)^*a^* + a^*) \\
 & = & (a+b)^*a + 1 + a\backslash a^* \\
 & = & (a+b)^*a + 1 + a^* \\
 & = & (a+b)^*a + 1 + a^*a + 1 \\
 & = & (a+b)^*a + 1 \quad \text{wegen } a^* \leq (a+b)^* \\
ab\backslash r & = & b\backslash(a\backslash r) = b\backslash((a+b)^*a + (bb)^*a^*) \\
 & = & b\backslash((a+b)^*a) + b\backslash(bb(bb)^*a^*) + b\backslash a^* \\
 & = & (a+b)^*a + b\backslash a + b(bb)^*a^* + 0 \\
 & = & (a+b)^*a + b(bb)^*a^* \\
ba\backslash r & = & a\backslash(b\backslash r) = (a+b)^*a + a\backslash a \\
 & = & (a+b)^*a + 1 = aa\backslash r \\
bb\backslash r & = & b\backslash(b\backslash r) = (a+b)^*a + b\backslash a \\
 & = & (a+b)^*a = b\backslash r \\
aaa\backslash r & = & a\backslash(aa\backslash r) \\
 & = & a\backslash((a+b)^*a + 1) \\
 & = & (a+b)^*a + a\backslash a + a\backslash 1 \\
 & = & (a+b)^*a + 1 = aa\backslash r \\
aab\backslash r & = & b\backslash(aa\backslash r) \\
 & = & b\backslash((a+b)^*a + 1) \\
 & = & (a+b)^*a + b\backslash a + b\backslash 1 \\
 & = & (a+b)^*a = b\backslash r \\
aba\backslash r & = & a\backslash(ab\backslash r) = a\backslash((a+b)^*a + b(bb)^*a^*) \\
 & = & a\backslash(a+b)^*a \\
 & = & (a+b)^*a + 1 = aa\backslash r \\
abb\backslash r & = & b\backslash(ab\backslash r) = b\backslash((a+b)^*a + b(bb)^*a^*) \\
 & = & (a+b)^*a + b\backslash a + (bb)^*a^* \\
 & = & (a+b)^*a + (bb)^*a^* = a\backslash r
\end{array}
$$

Es gibt also nur fünf verschiedene $w\backslash r$, und diese führen zum Automaten



Die Endzustände sind die angegebenen, da $a, aba \in L$ und $\epsilon, b, ab \notin L$.

(iii) Für $L = \{ a^n b^n \mid n \in I\!N \}$ erhält man

$$a \backslash L = a \backslash (\{\epsilon\} \cup aLb), = a \backslash (aLb) = Lb \tag{1}$$

und induktiv aus $a^n \backslash L = Lb^n$ dann

$$a^{n+1} \backslash L = a \backslash (a^n \backslash L) = a \backslash (Lb^n) = a \backslash (\{b^n\} \cup aLbb^n) = Lb^{n+1}.$$

Diese Sprachen $\{ Lb^n \mid n \in I\!N \}$ sind alle von einander verschieden, denn für $k < n$ ist $b^n \in Lb^n - Lb^k$ wegen $\epsilon \in L$.

Beachte, daß es im Automaten für die unendliche, reguläre Sprache in (b) echte Zyklen gibt, während der Automat für die endliche Sprache in (a) nur den "künstlichen" Zyklus bei $a \backslash \emptyset = \emptyset = b \backslash \emptyset$ gibt. $\triangle$