Lecture 8: Rule extraction and training

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Statistical Machine Translation

Stuttgart — January 13, 2012

Lecture 8

Last time

- Basic setup
- Tree automata

This time

- Synchronous grammars (tree transducers)
- Rule extraction
- Weight training (EM)

Contents

Weight structure

Definition

Commutative semiring $(C, +, \cdot, 0, 1)$ if

- $(C, +, 0)$ and $(C, \cdot, 1)$ commutative monoids
- $\cdot$ distributes over finite (incl. empty) sums

Example

- BOOLEAN semiring $(\{0, 1\}, \max, \min, 0, 1)$
- Semiring $(\mathbb{R}_{\geq 0}, +, \cdot, 0, 1)$ of probabilities
- Tropical semiring $(\mathbb{N} \cup \{\infty\}, \min, +, \infty, 0)$
- Any field, ring, etc.

Most of the lecture: BOOLEAN semiring (rule extraction) or probabilities

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Computing in semirings

Distributivity

$$a \cdot \left(\sum_{i \in I} a_i \right) = \sum_{i \in I} (a \cdot a_i)$$

Absorption

$0 \cdot a = 0$

Questions

- $1 + a = 1$?
- $a + (\prod_{i \in I} a_i) = \prod_{i \in I} (a + a_i)$?
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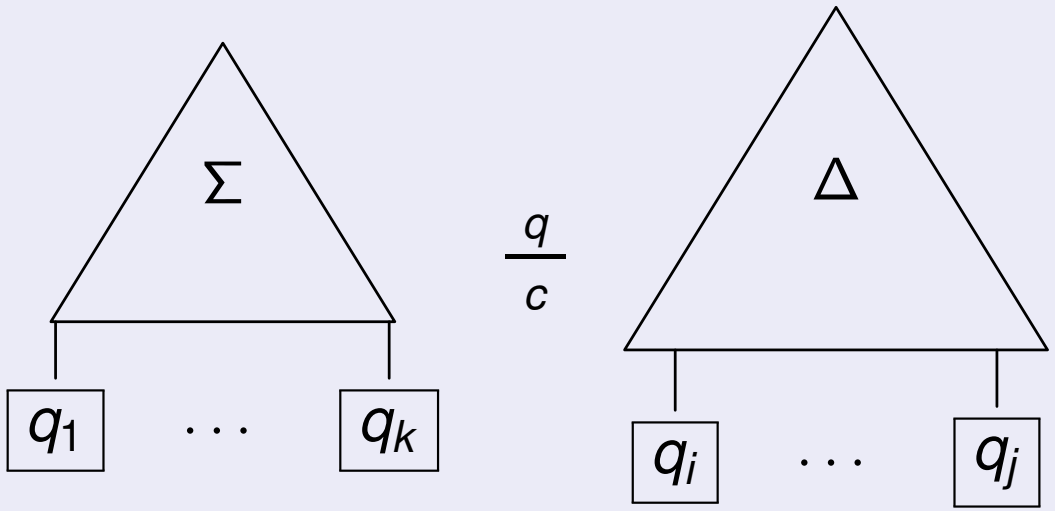
Syntax

Definition (Arnold, Dauchet 1976, Graehl, Knight 2004)

Extended top-down tree transducer (XTOP) $M = (Q, \Sigma, \Delta, I, R)$ with finitely many rules

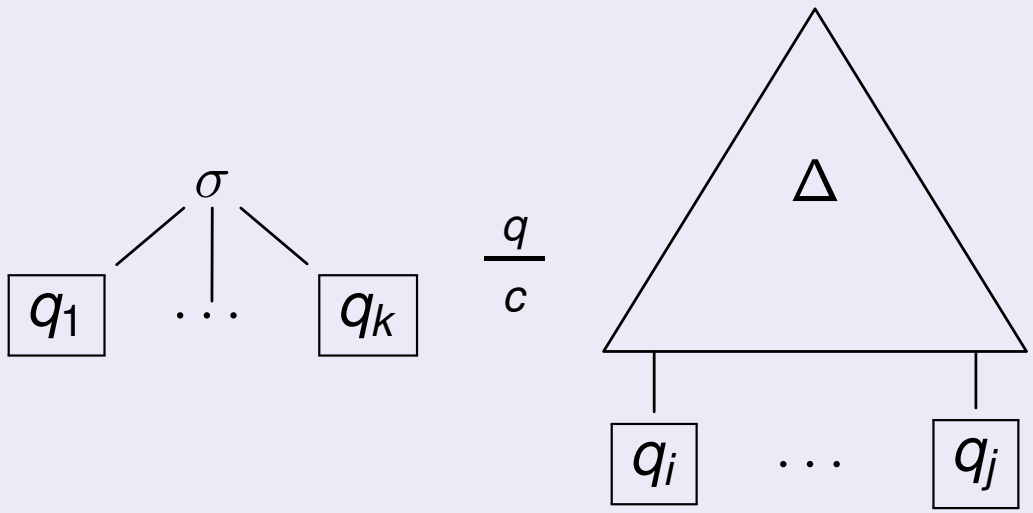


- $q, q_1, \ldots, q_k \in Q$ are states
- $c \in C$ (weight)

Syntax (cont'd)

Definition (Rounds 1970, Thatcher 1970)

Top-down tree transducer (TOP) if all rules



Remarks

- TOP well-studied device
- XTOP much less so, but gained a lot of traction

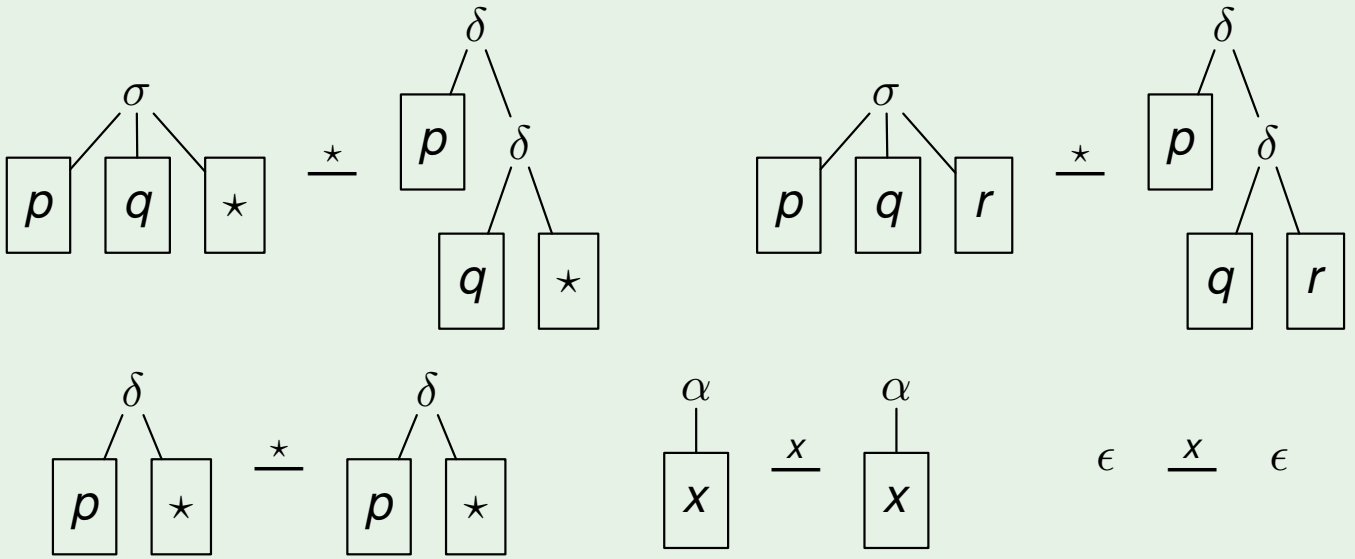
Extended Top-down Tree Transducer

Definition

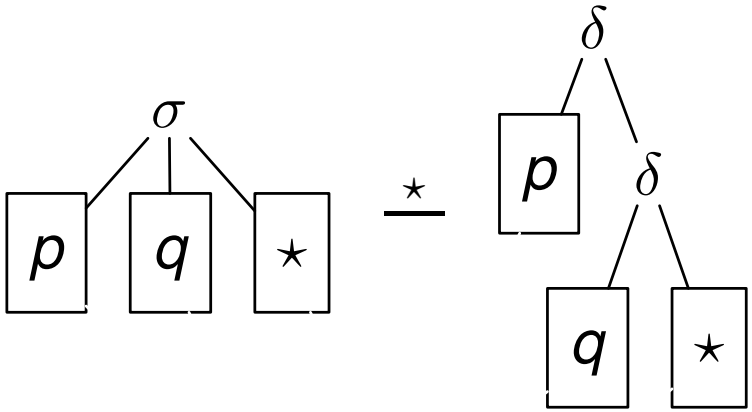
Each rule now has an input and an output side, which are both full trees (not just a single symbol followed by states)

Example

We ignore weights for the moment.



Link Structure



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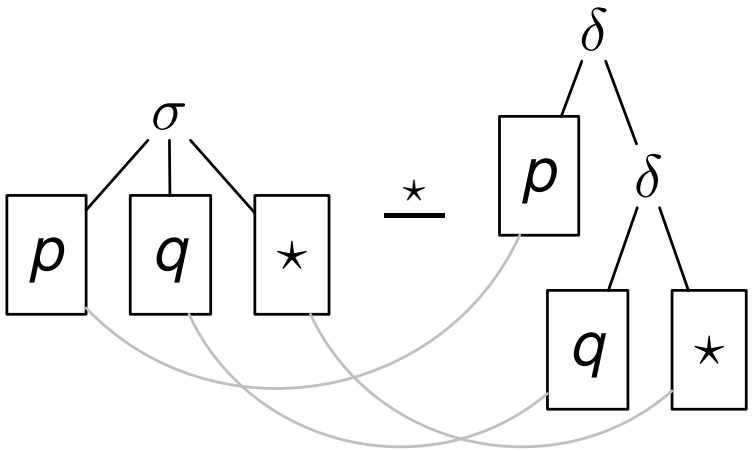

Derivation

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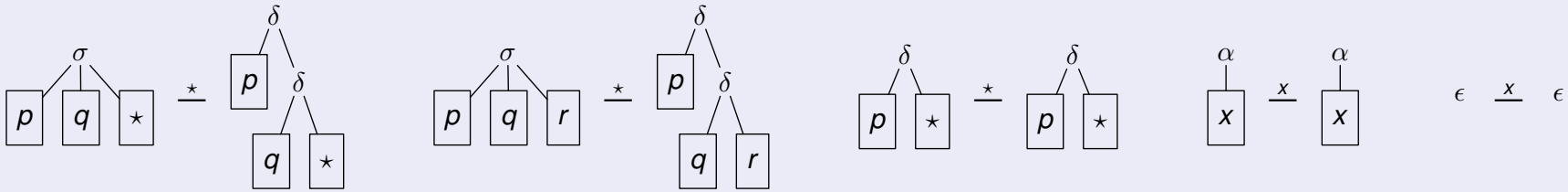

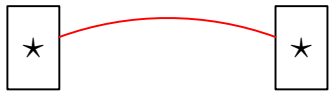

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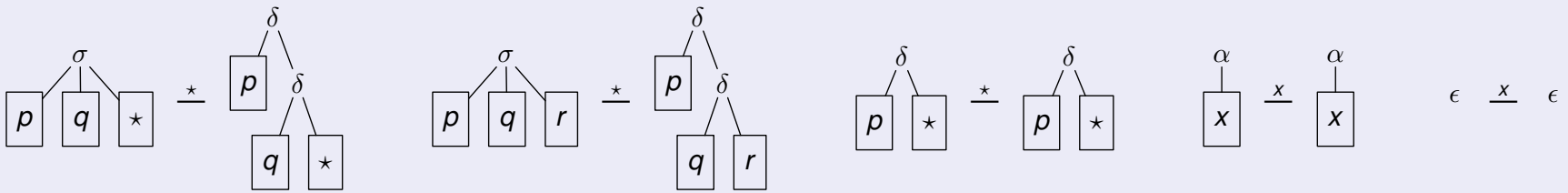

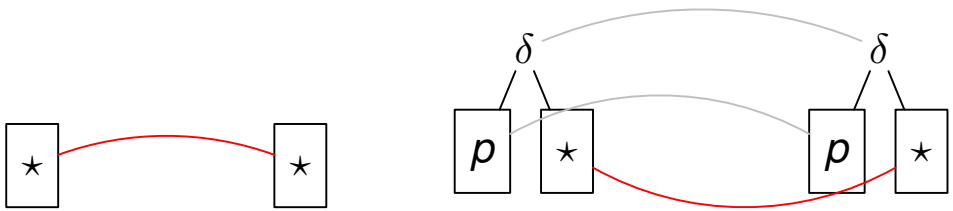

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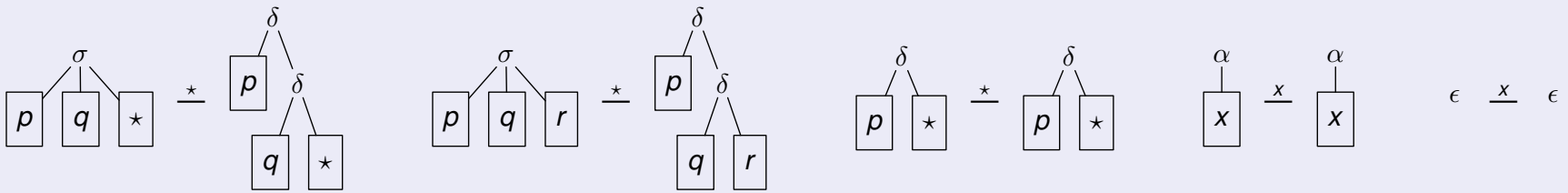

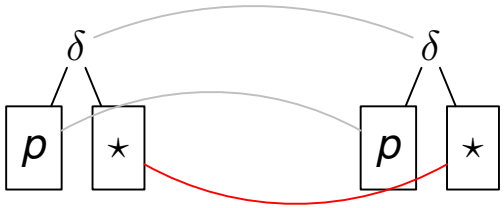

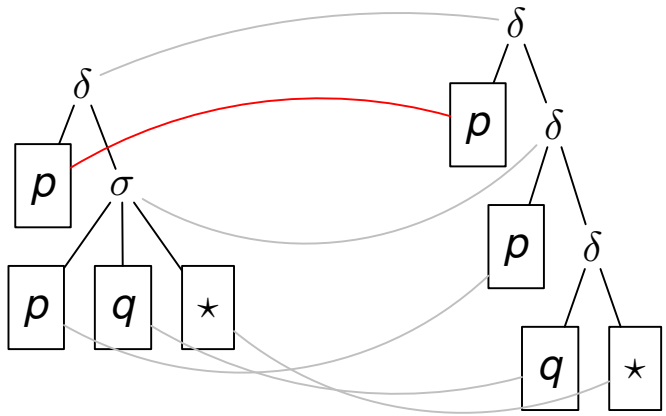

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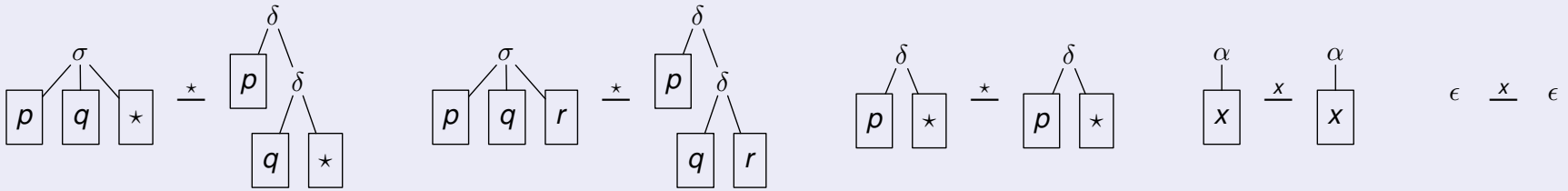

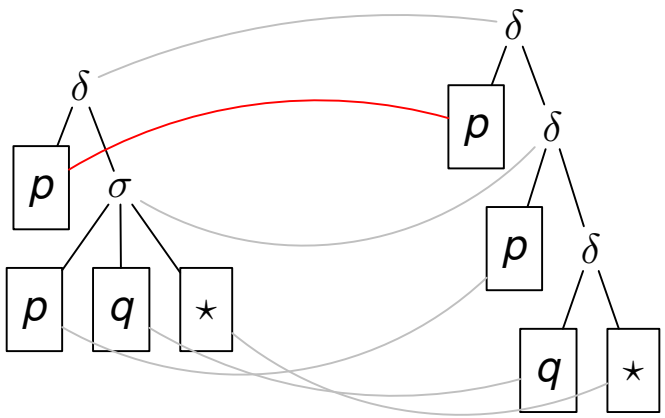

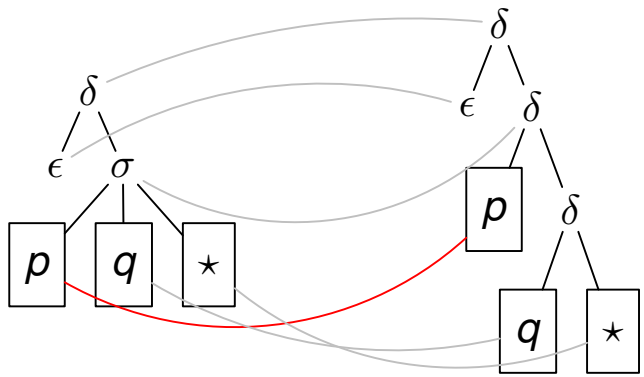

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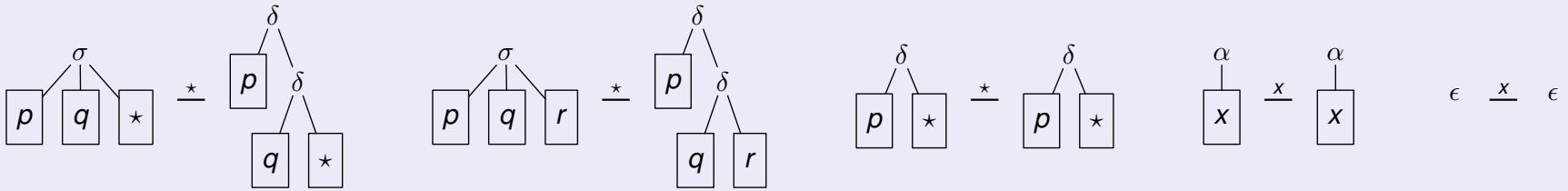

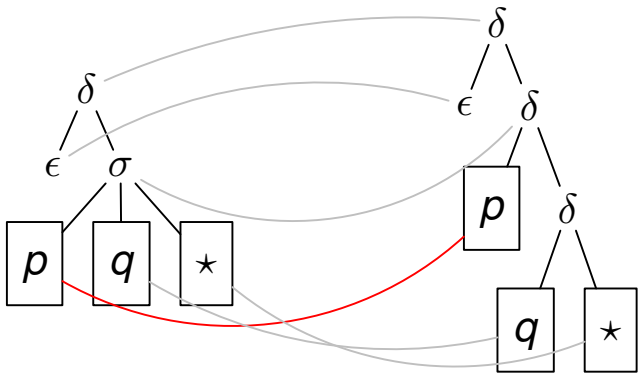

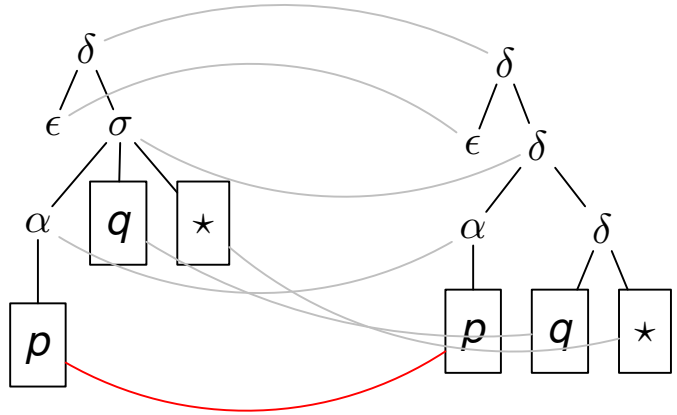

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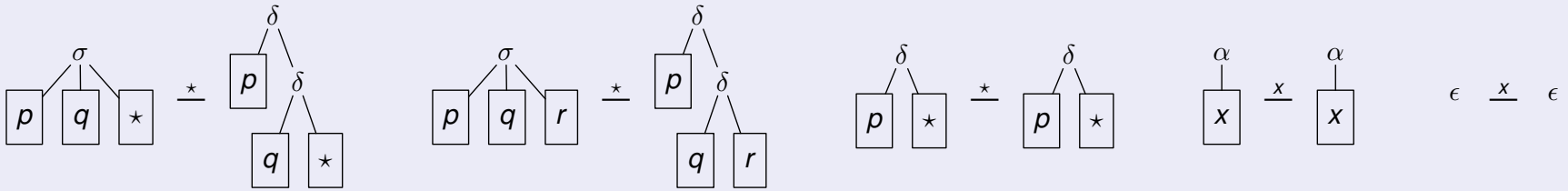

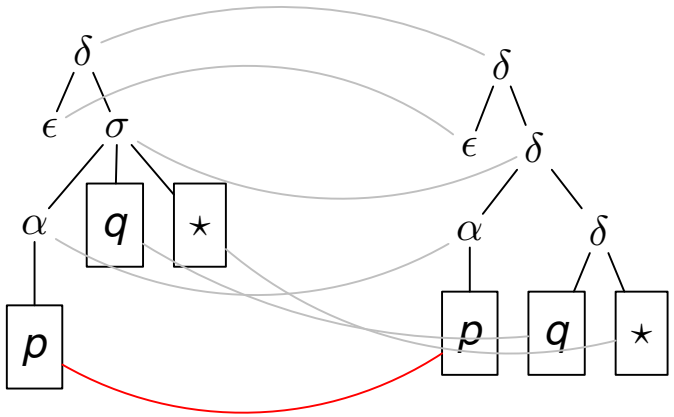

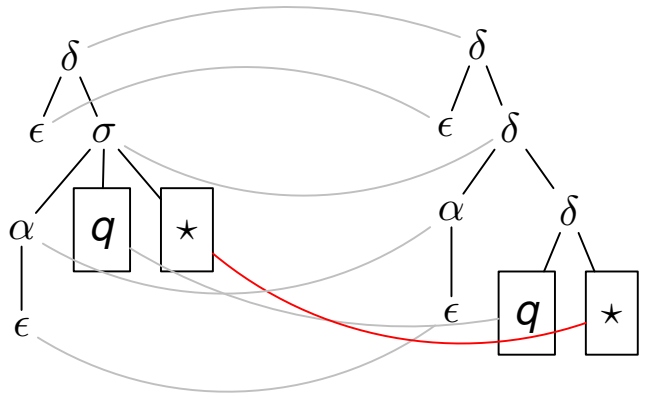

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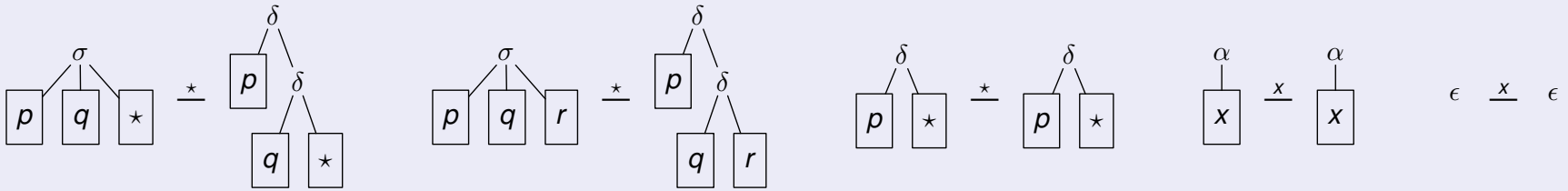

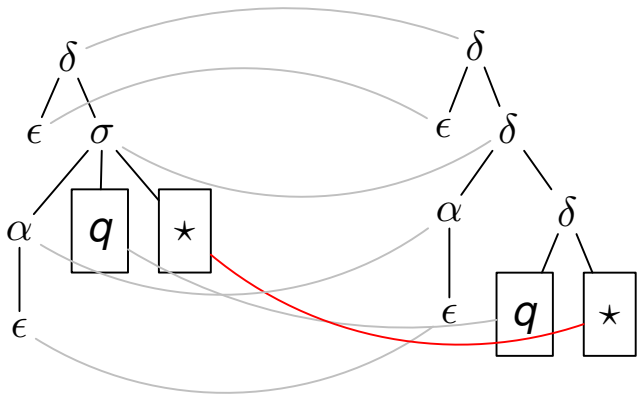

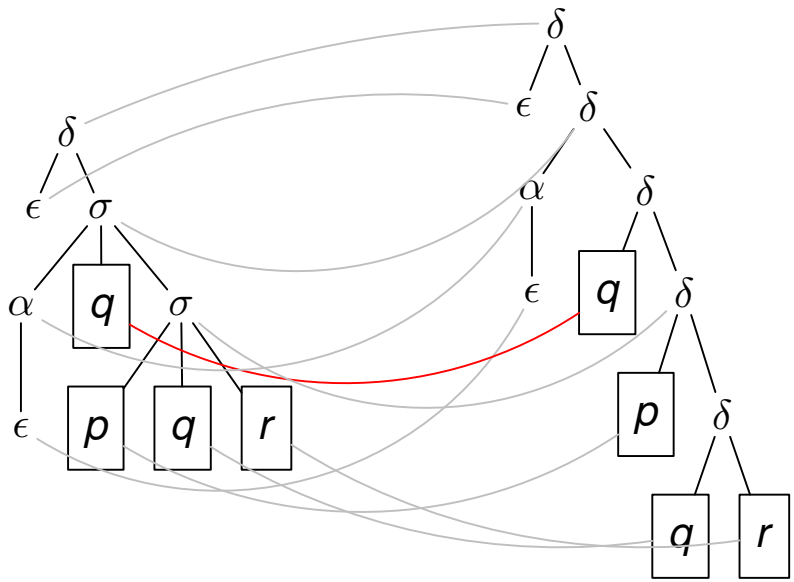

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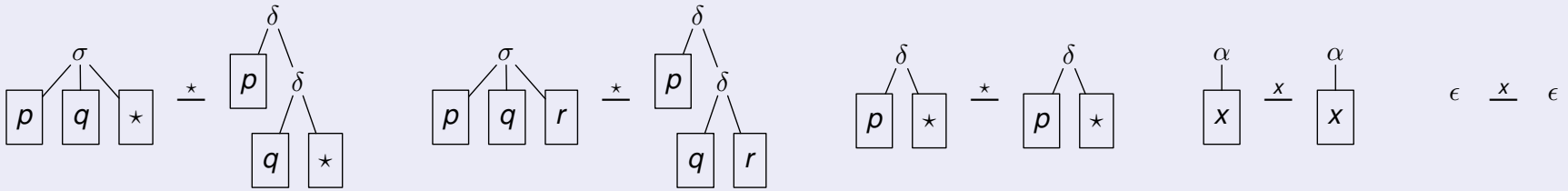

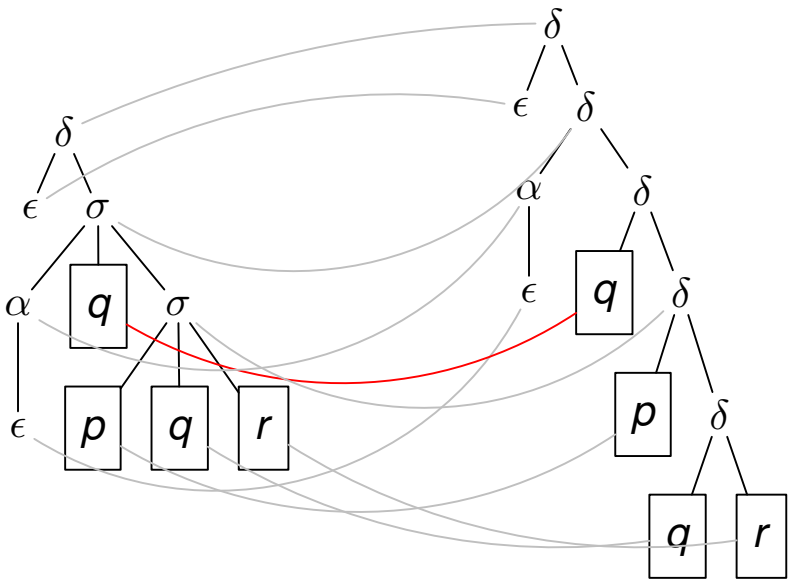

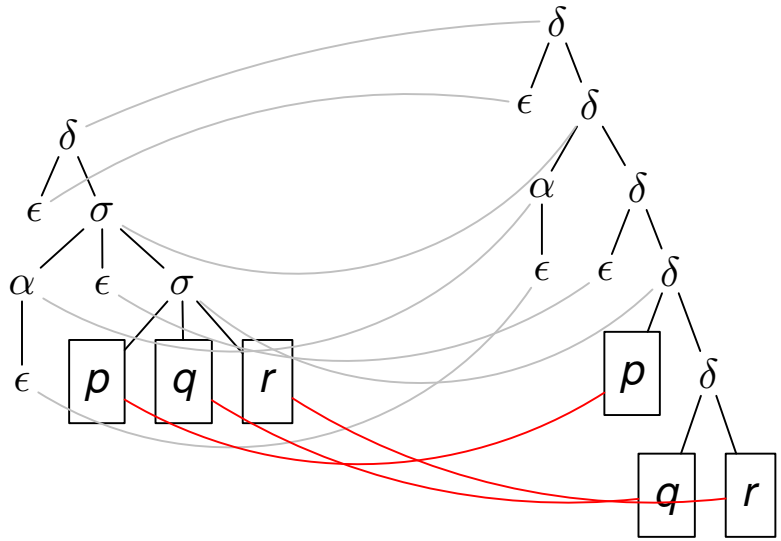

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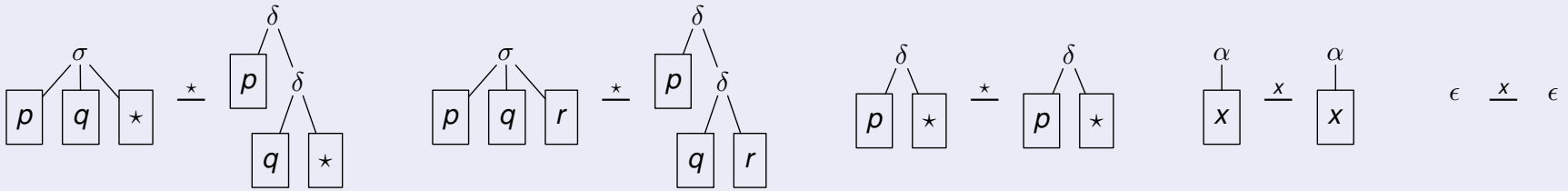
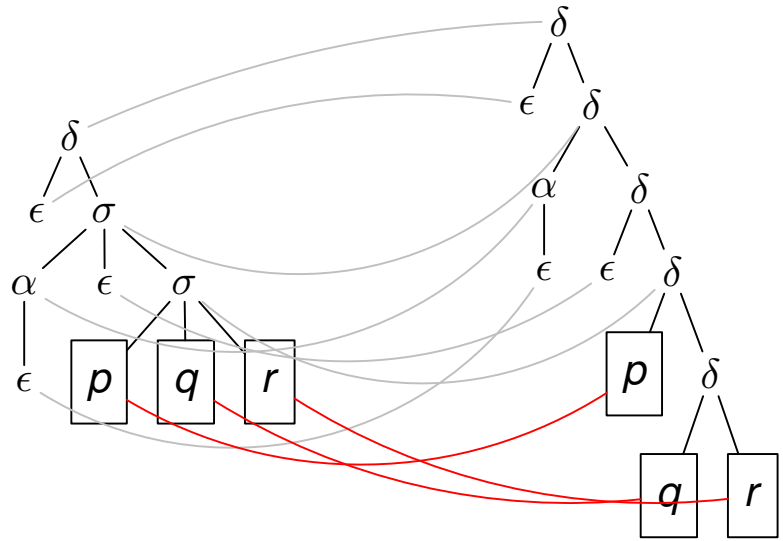
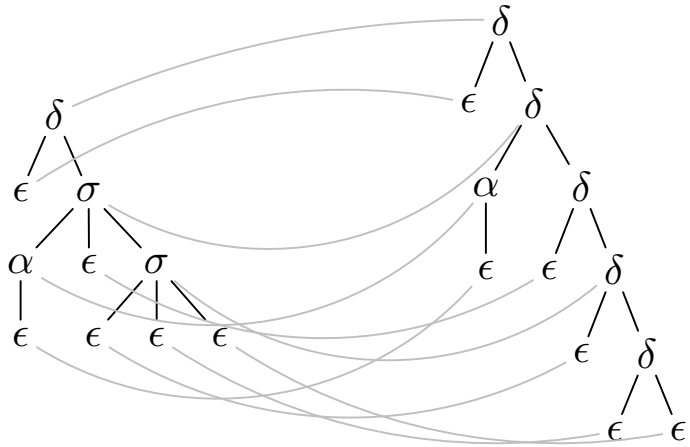
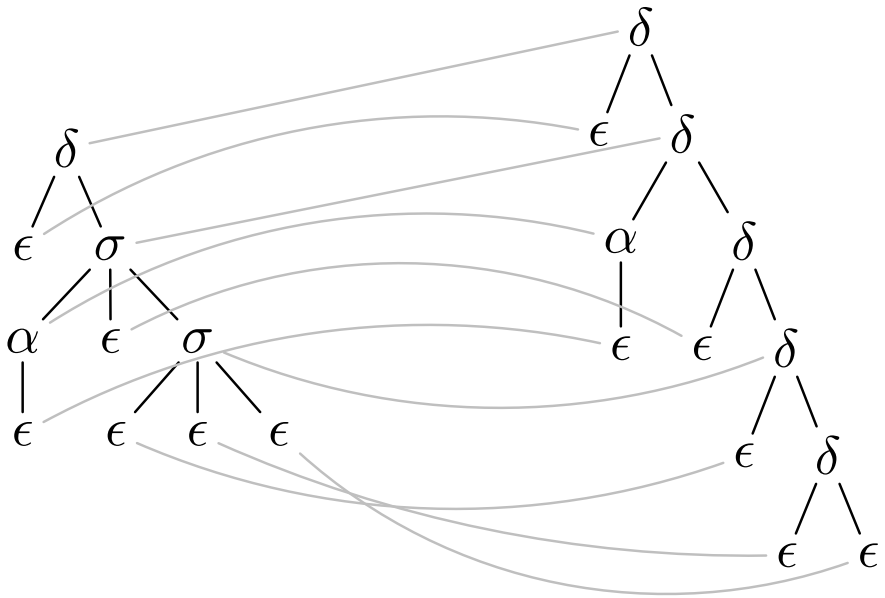






Semantics of XTOP



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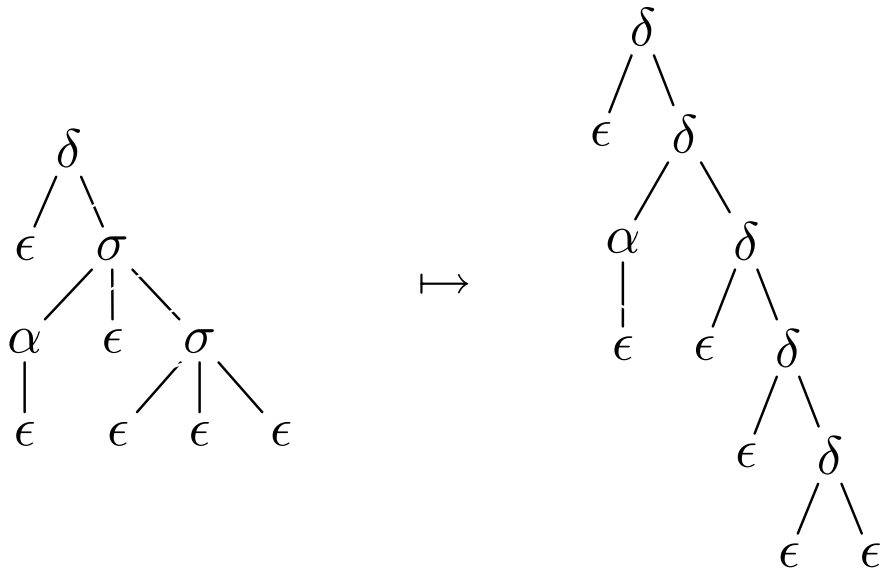

Comprehension

Question

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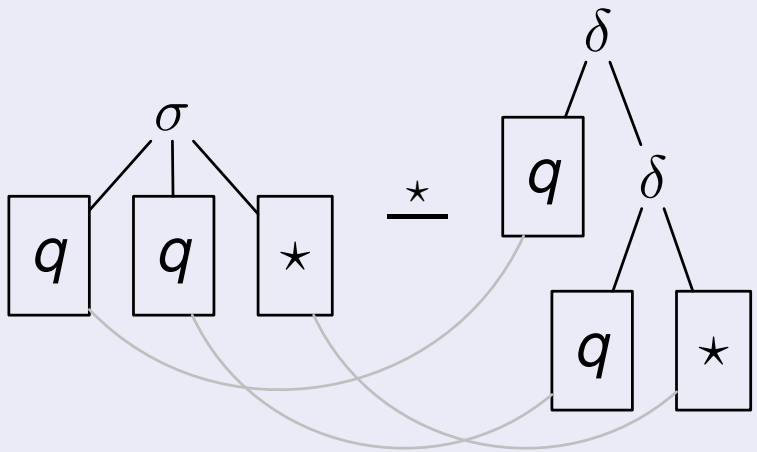

Can we implement it with an XTOP?

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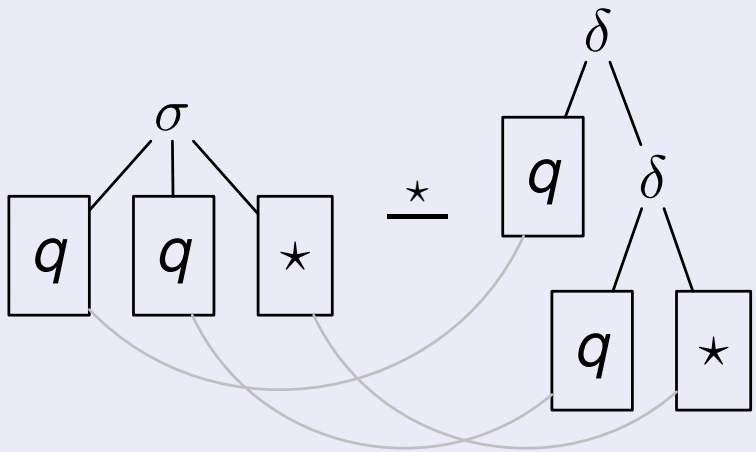


Can we implement it with an XTOP?

Answer

Yes. Copy all rules for q with a new head state p.
Replace one q consistently by p.

Contents

Rule extraction

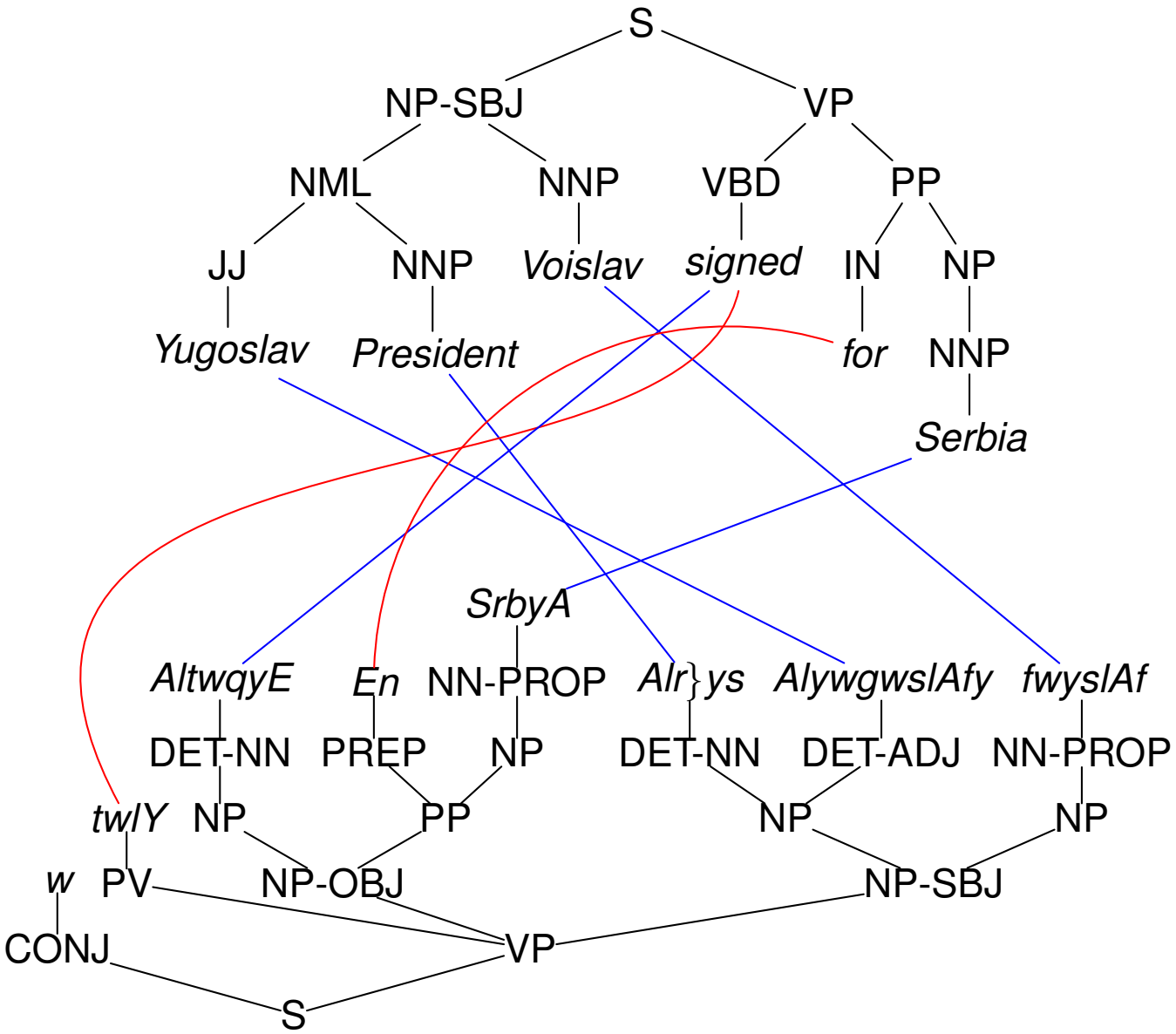
Goal

- automatic rule generation for XTOP
- only “useful” rules

Requirements

- bi-text
- parses for both sides
- string alignment (GIZA++)

Rule extraction



Rule extraction

Additional real requirements

- good alignments (use alignment correction / completion)
- side conditions (typical: POS never without lexical item)

Approach

- bottom-up procedure
- extend alignments to parse trees
- extract aligned subtrees

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Two subtrees *t* and *u* are consistently aligned if all alignments from nodes of *t* align to nodes inside *u* and vice versa.

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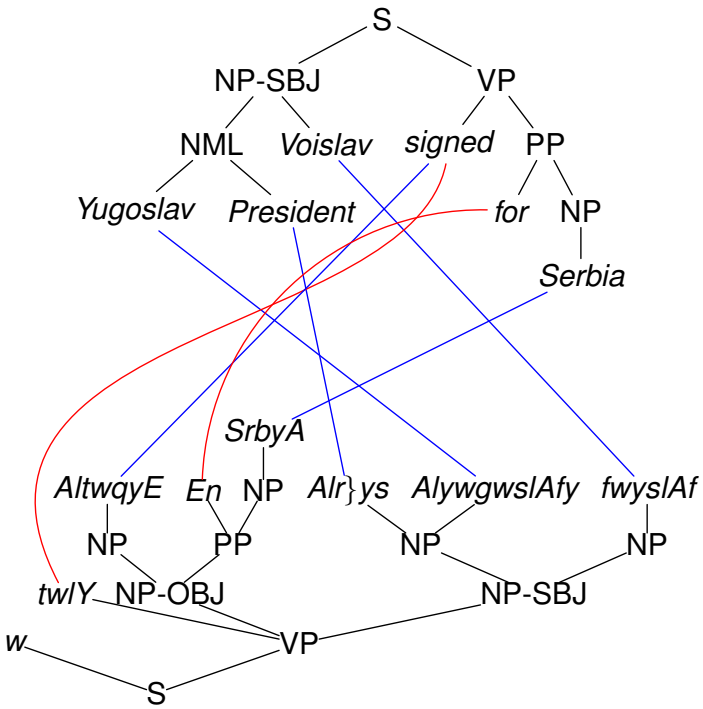

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- Identify maximal subtree of aligned nodes
- Identify subtree of nodes aligned to aligned nodes, etc.
- Extract rule and leave state
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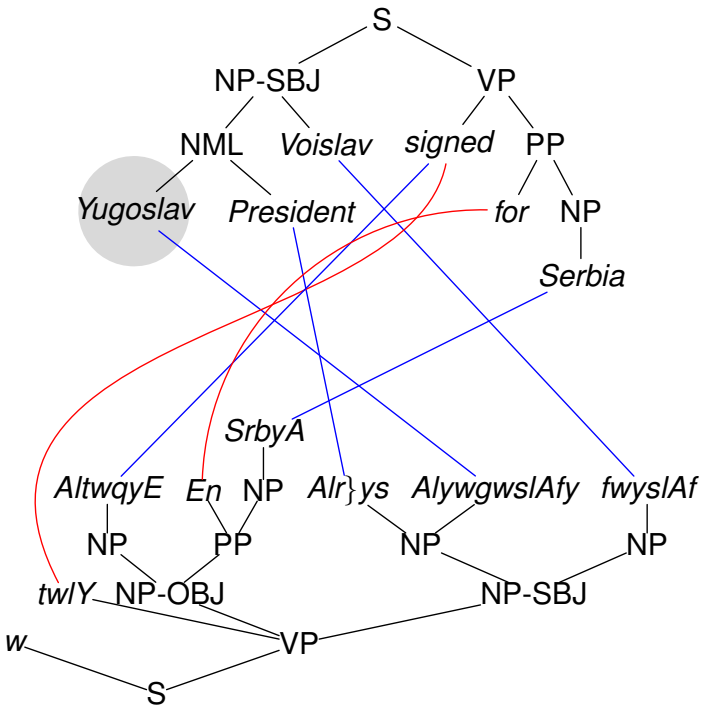
Rule extraction



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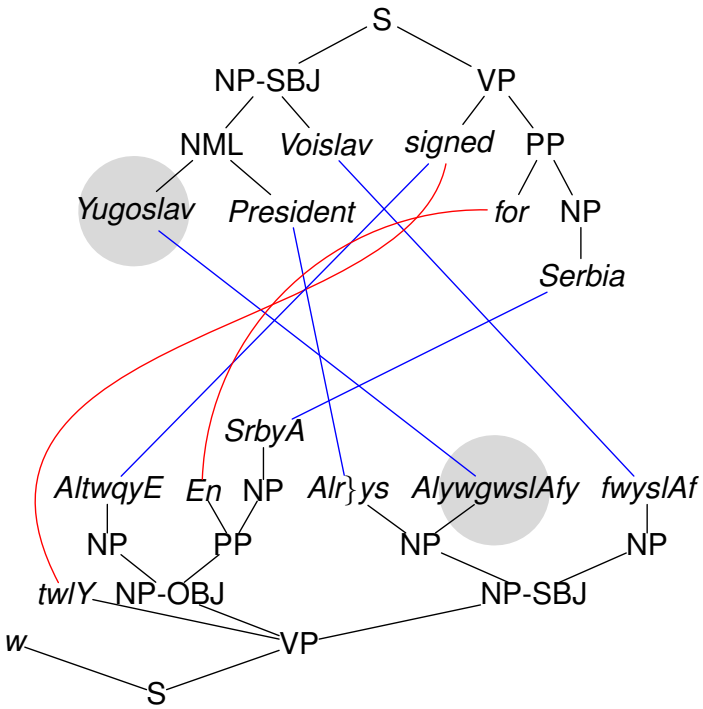

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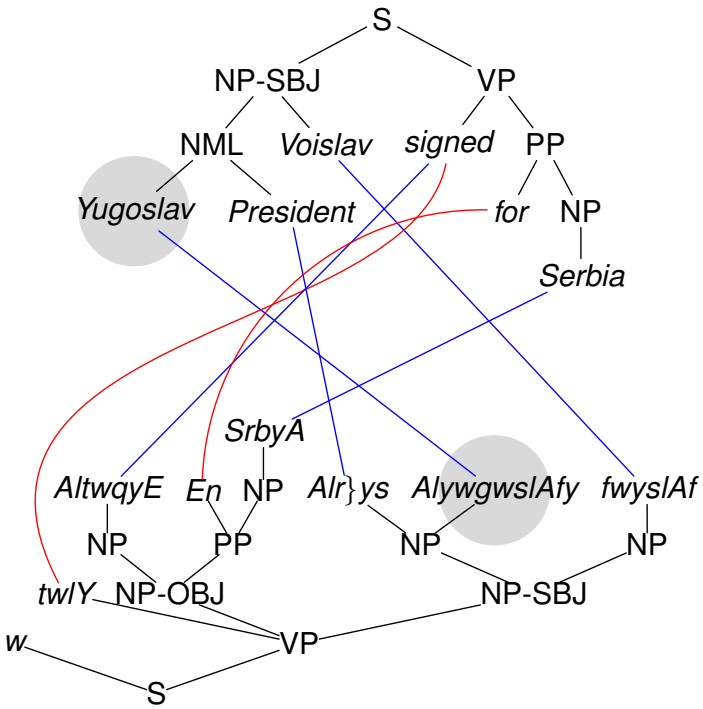

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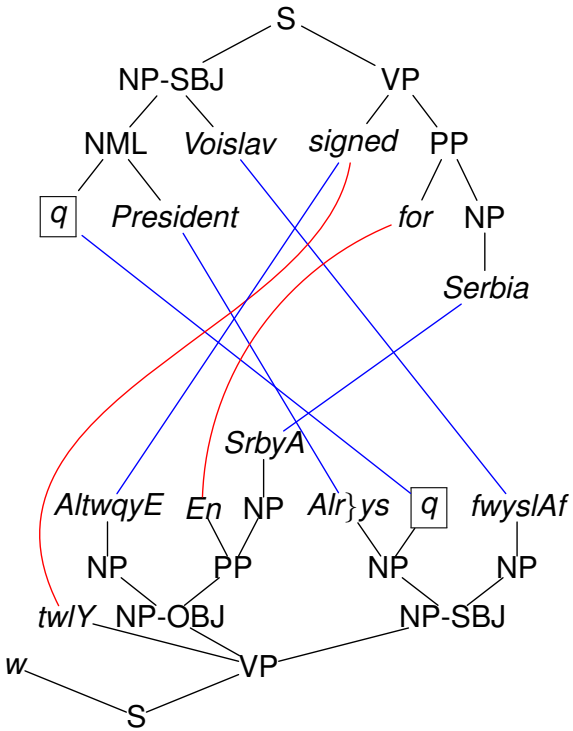

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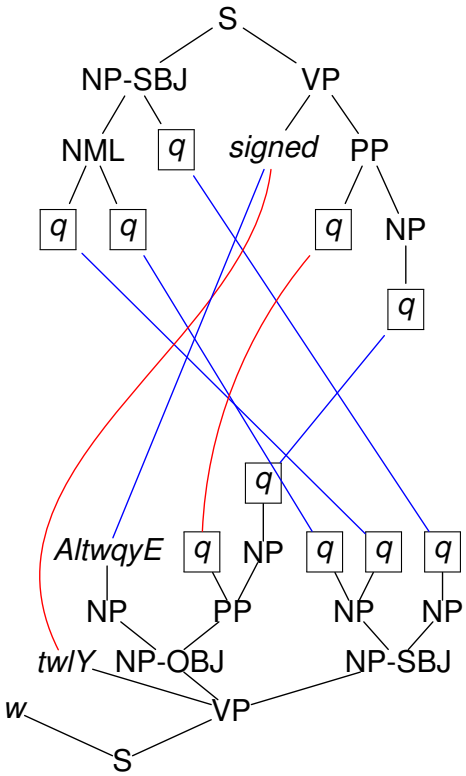

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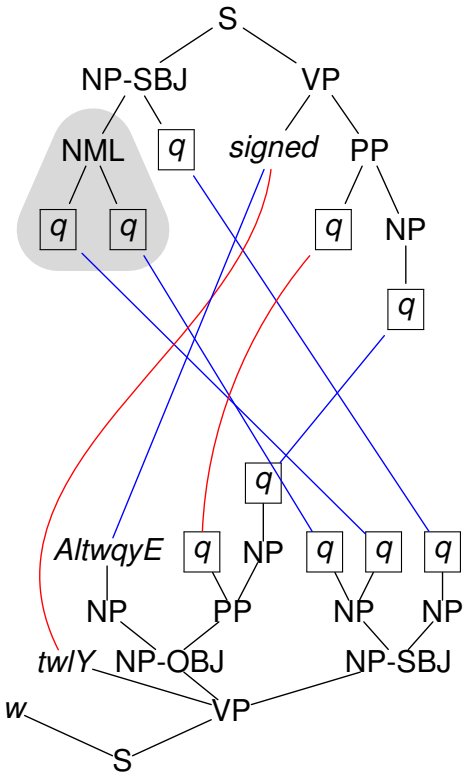
$$\textit{President} \xrightarrow{q} \textit{Alr}\}\textit{ys}$$

$$\textit{Voislav} \xrightarrow{q} \textit{fwyslAf}$$

$$\textit{for} \xrightarrow{q} \textit{En}$$

$$\textit{Serbia} \xrightarrow{q} \textit{SrbyA}$$

Rule extraction



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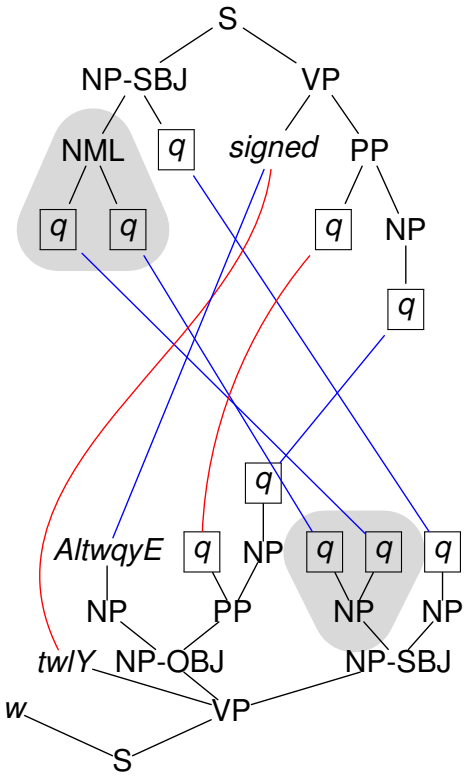

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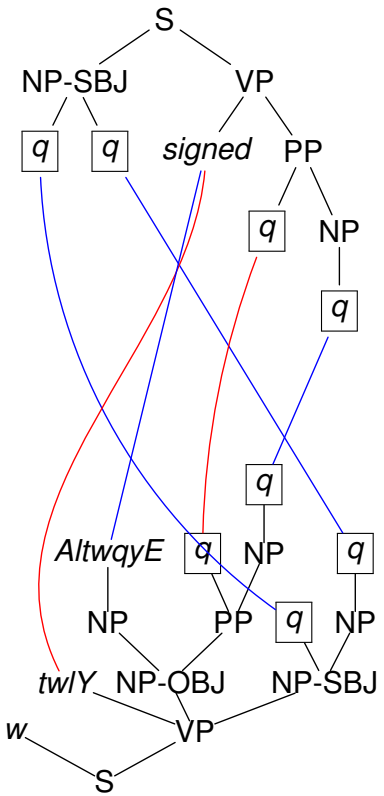
$$\mathrm{NML}(q_1, q_2) \xrightarrow{q} \mathrm{NP}(q_2, q_1)$$

Rule extraction



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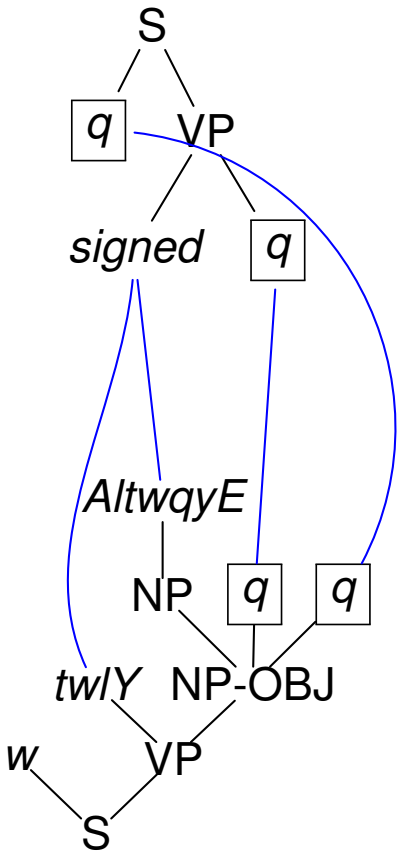

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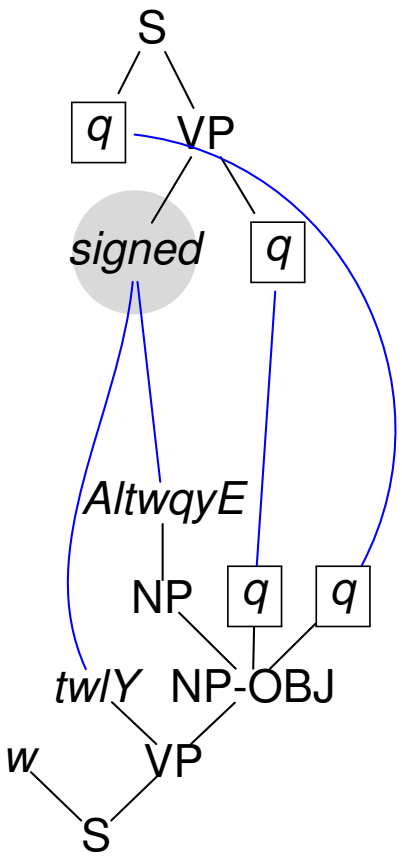

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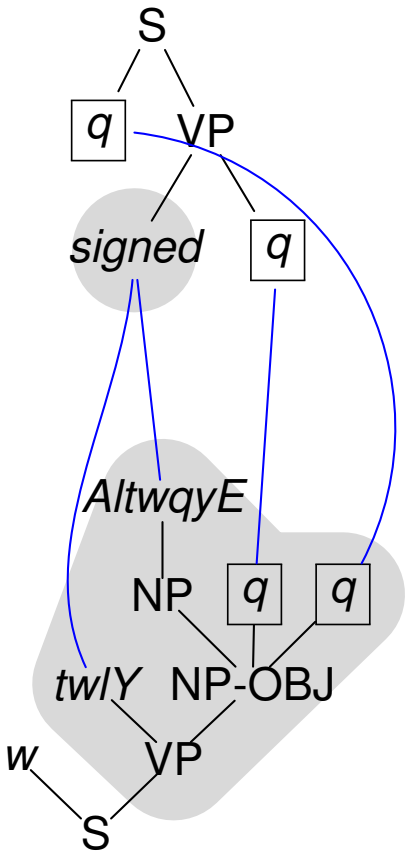

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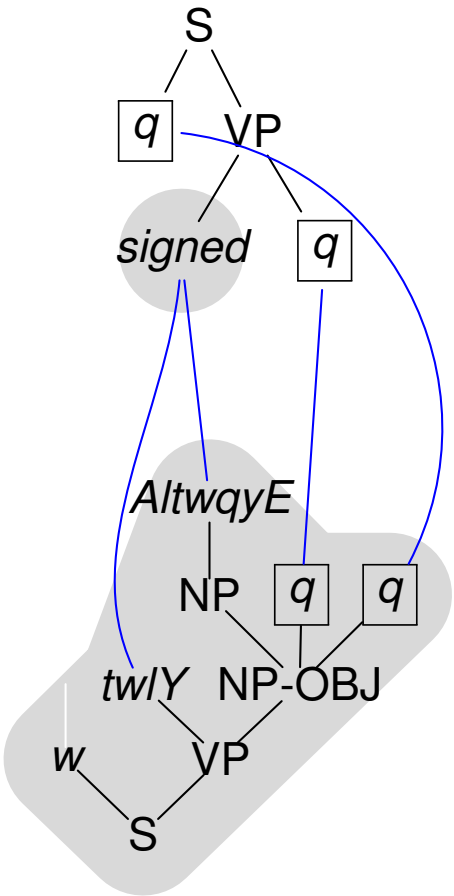

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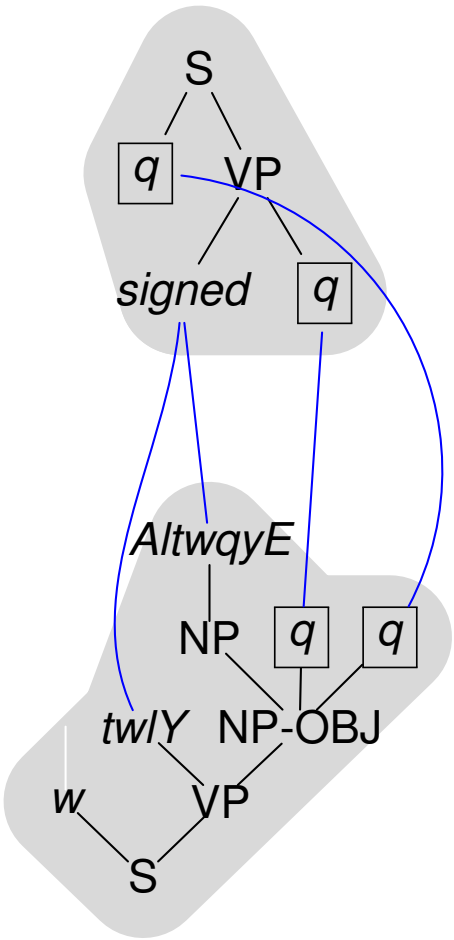

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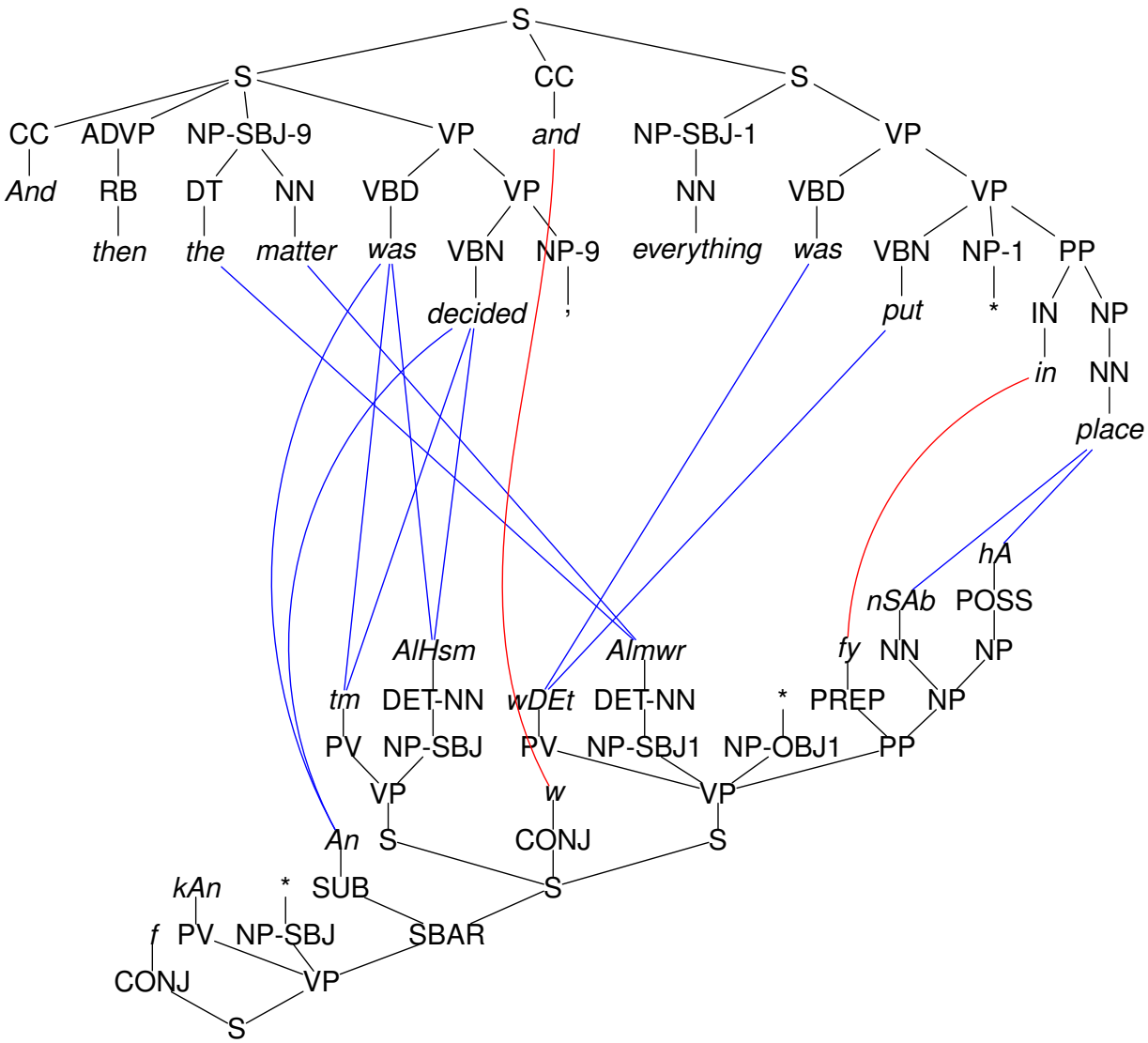
Rule extraction exercise



Contents

The problem

Input

- weighted bi-parsed bi-text T (our training corpus)
- unweighted XTOP M (the extracted XTOP)

Goal

- Obtain rule weight for each rule

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Weighted XTOP

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- weights in a derivation are multiplied
- weights of derivations are summed

Derivation strategy

- given a partial derivation
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- Left-most derivation (deterministic)
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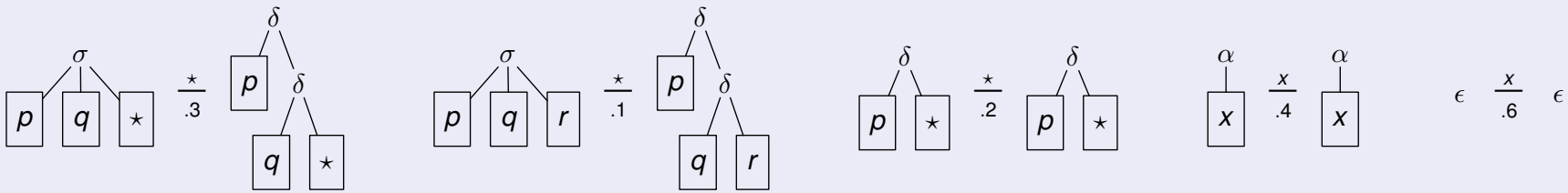
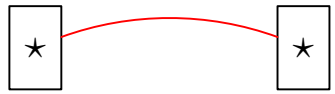
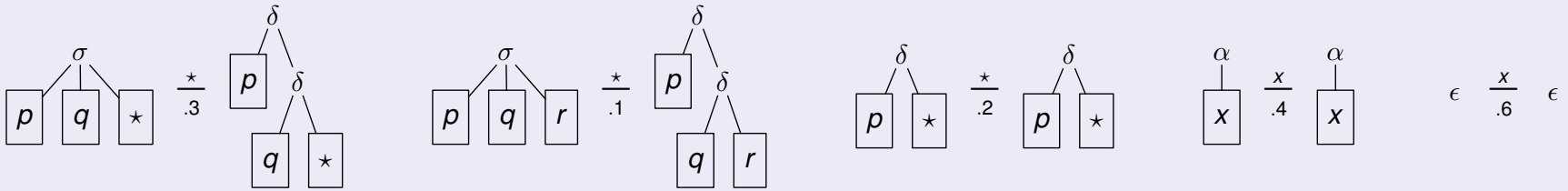


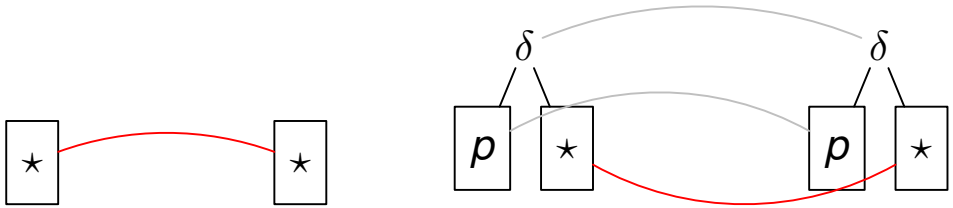


Derivation weight:

Derivation

Rules

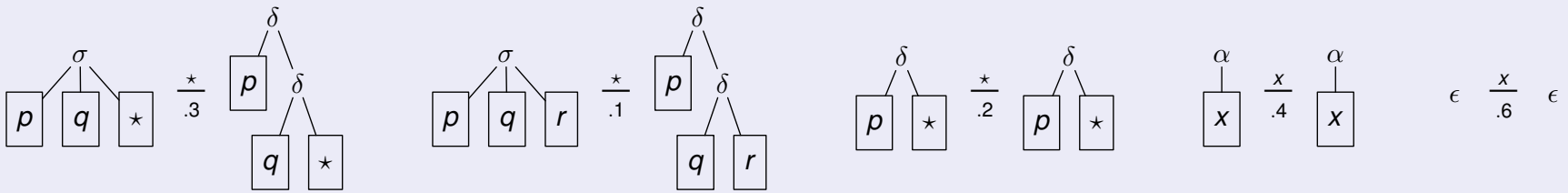


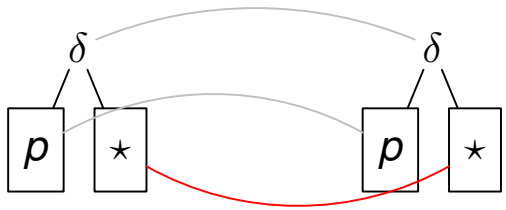


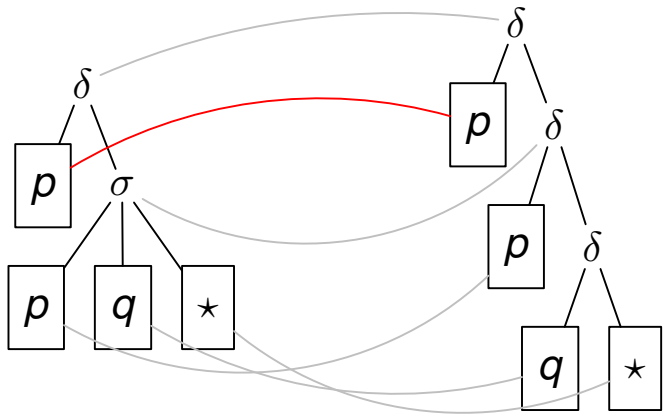
Derivation weight: 0.2

Derivation

Rules



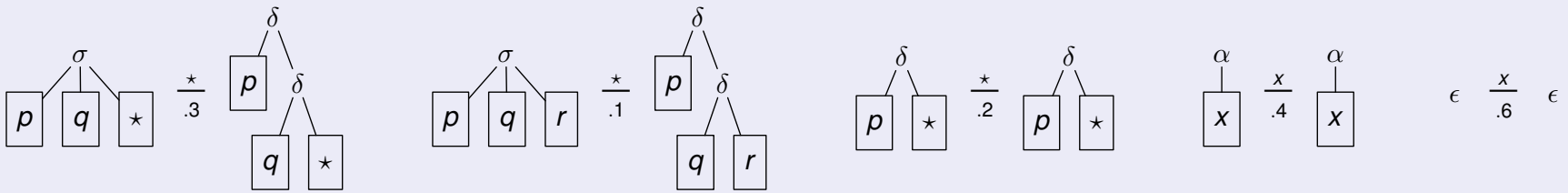


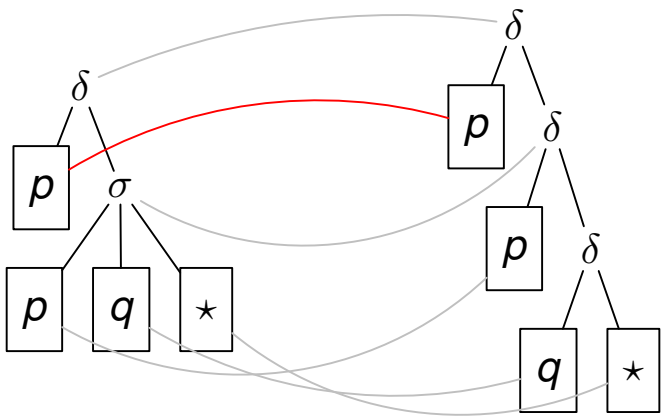


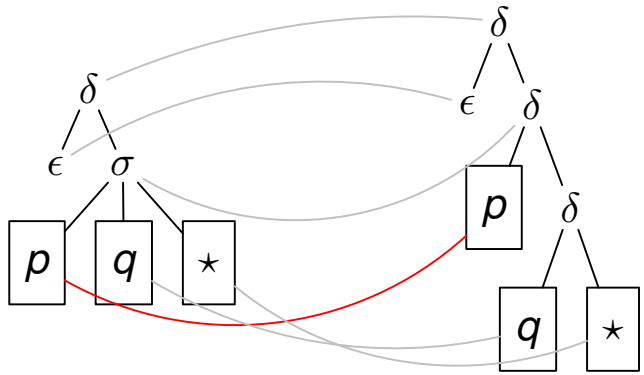
Derivation weight: $0.2 \cdot 0.3$

Derivation

Rules



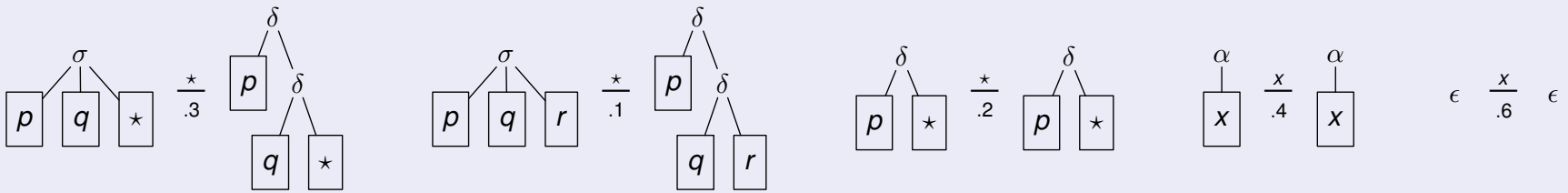


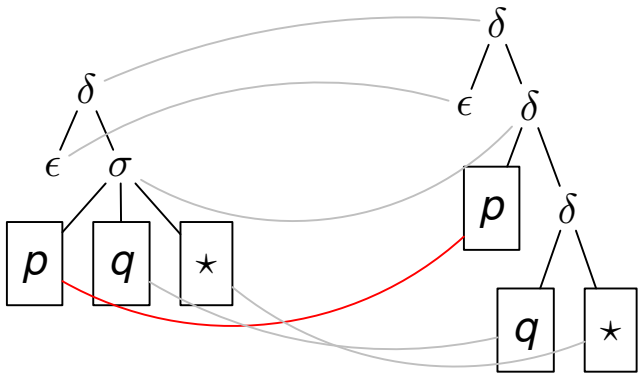


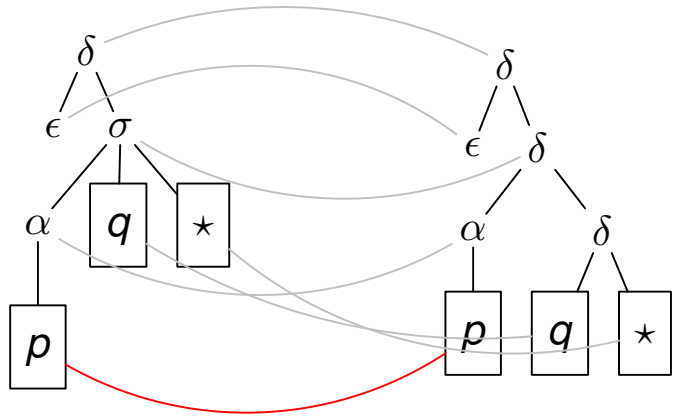
Derivation weight: $0.2 \cdot 0.3 \cdot 0.6$

Derivation

Rules



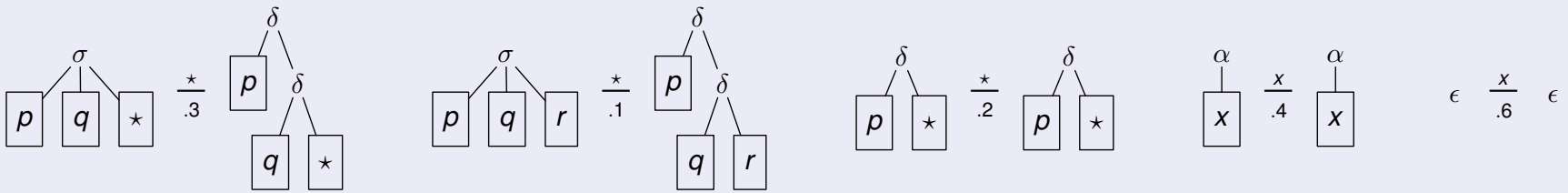


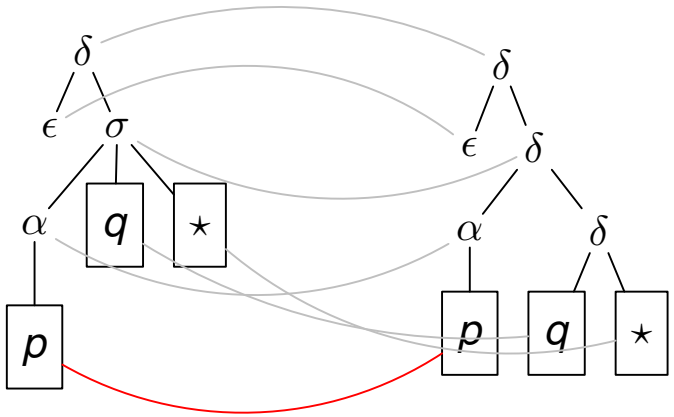


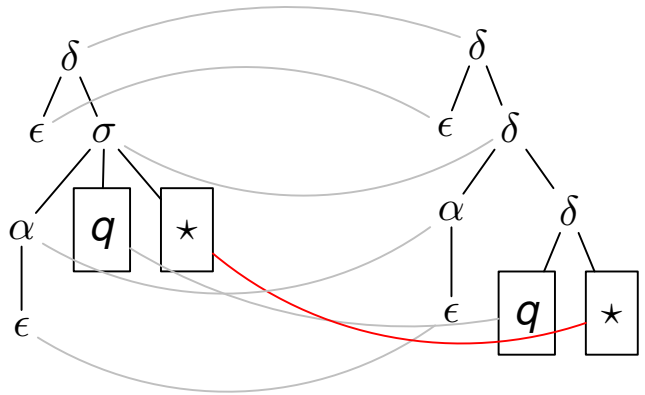
Derivation weight: $0.2 \cdot 0.3 \cdot 0.6 \cdot 0.4$

Derivation

Rules



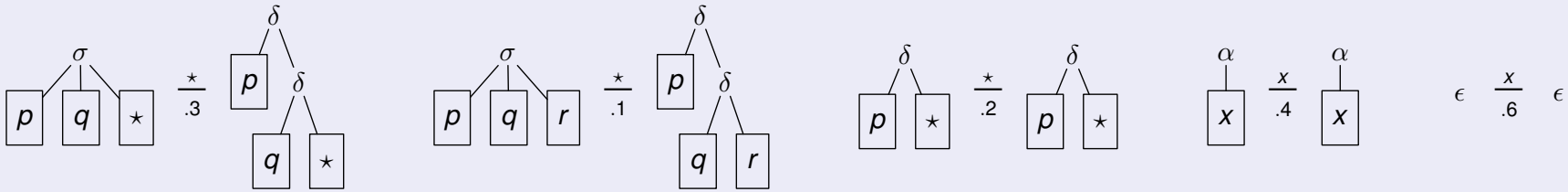


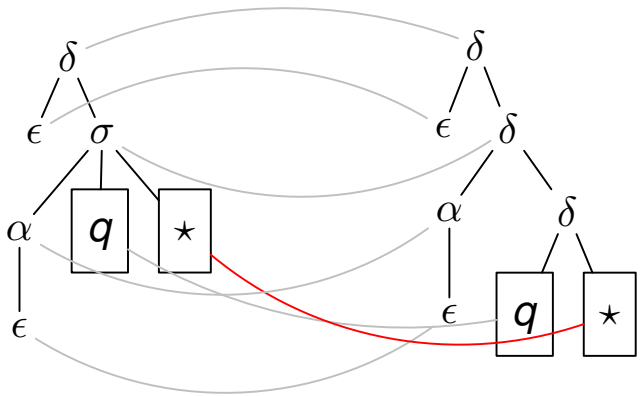


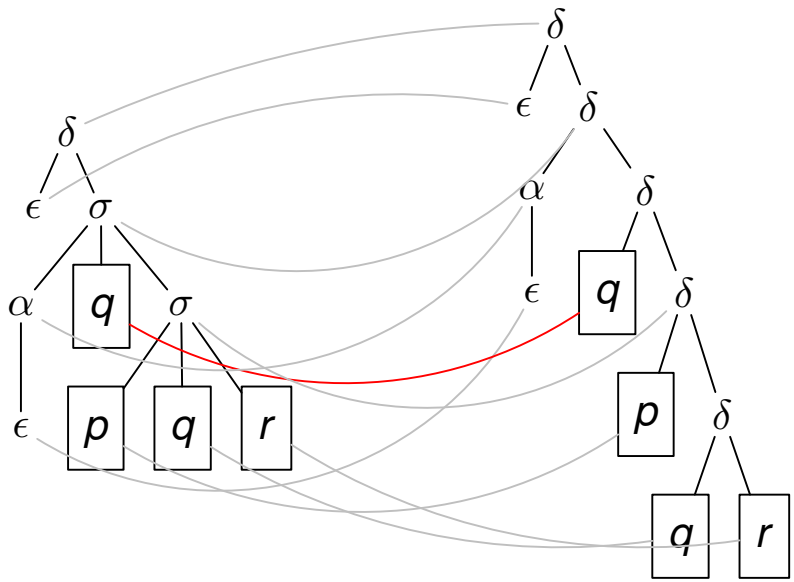
Derivation weight: $0.2 \cdot 0.3 \cdot 0.6 \cdot 0.4 \cdot 0.6$

Derivation

Rules



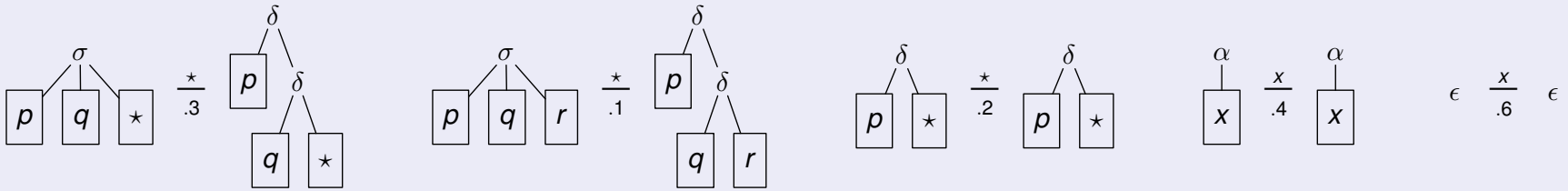


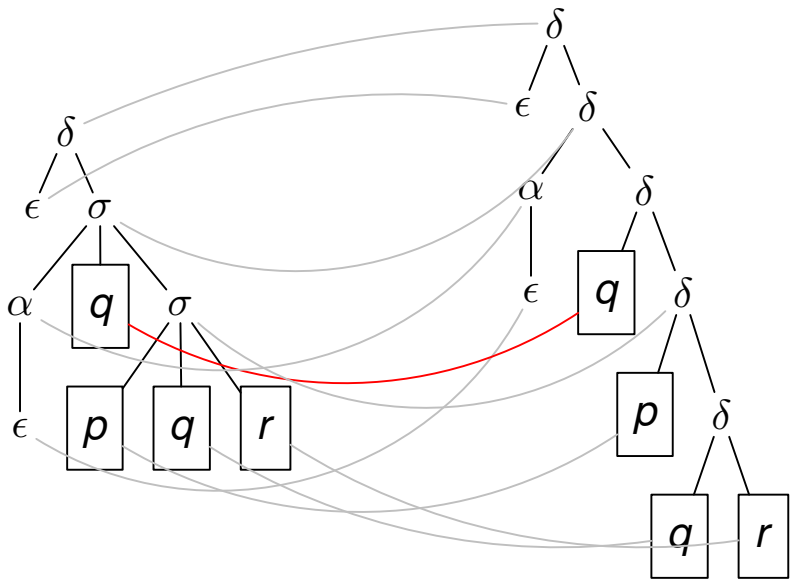


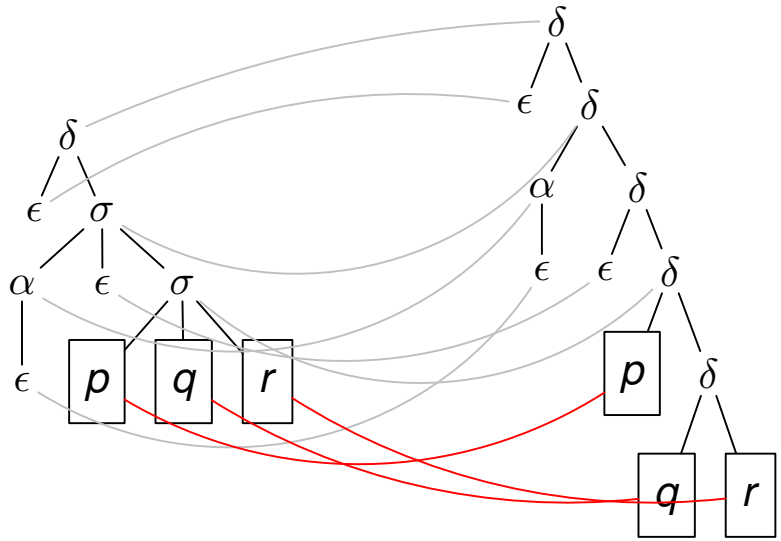
Derivation weight: $0.2 \cdot 0.3 \cdot 0.6 \cdot 0.4 \cdot 0.6 \cdot 0.1$

Derivation

Rules







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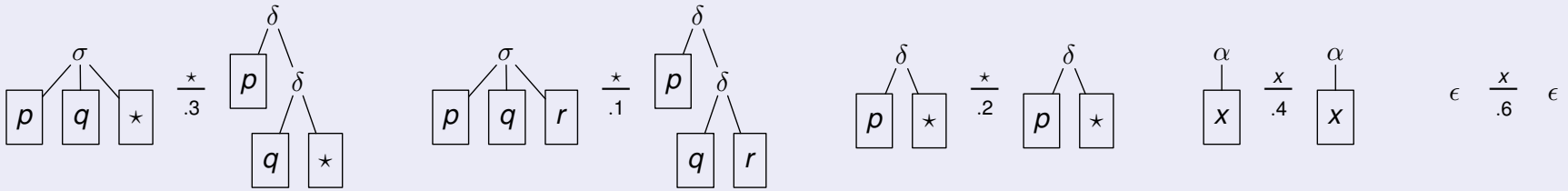

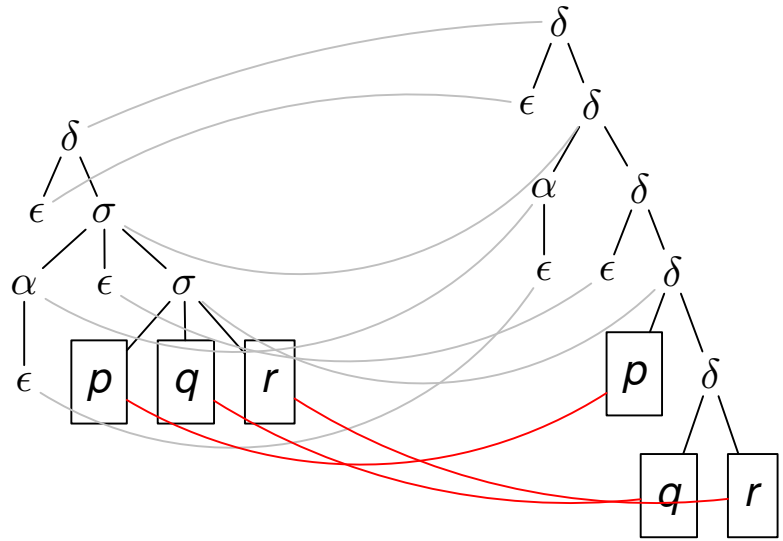

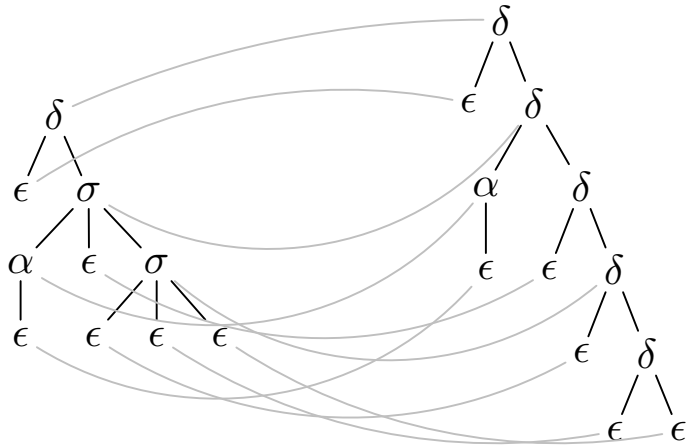

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Weighted Semantics of XTOP

$D_M(t, u)$: set of all (left-most) derivations for the tree pair (t, u)

$\mathrm{wt}(d) = \prod_{i=1}^{n} c_i$ for derivation $d = (q, q) \overset{c_1}{\Rightarrow} \cdots \overset{c_n}{\Rightarrow} (t, u)$ in $D_M(t, u)$

Definition

Computed transformation ($t \in T_\Sigma$ and $u \in T_\Delta$):

$$M(t, u) = \sum_{d \in D_M(t,u)} \mathrm{wt}(d)$$

EM Training

Basics

- EM = expectation maximization [Dempster, Laird, Rubin 1977]
- parameter improvement technique

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Expectation

$$c_r = E_{(t,u)\in T}\left[\frac{\sum_{d\in D_M(t,u)} \#_r(d)\cdot p(d)}{\sum_{d\in D_M(t,u)} p(d)}\right]$$

where $\#_r(d)$ is the number of occurrences of the rule r in d

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$$= E_{(t,u)\in T}\left[\frac{\sum_{d\in D_M(t,u)} \#_r(d)\cdot \mathrm{wt}(d)}{M(t,u)}\right]$$

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Maximization

$$\tau = \frac{w_r}{\sum_{r' \in R(r)} w_{r'}}$$

where $R(r)$ contains all rules that are for the same state as r

Question

Which probability distribution is modelled here?

EM Training

Remaining issues

- How to obtain initial weights?
- How to compute $D_M(t, u)$? all derivations for t and u
- How to compute the expectation?

Initial Weights

Possibilities

- uniform distribution on all rules for the same state
- uniform distribution based on state and “parseable” input (difficult)
- use normalized extraction counts

Derivations

- see last lecture
- can all be represented as weighted tree grammar
- often you can just try to enumerate them
  (finite language ⟶ trivial weighted tree grammar)

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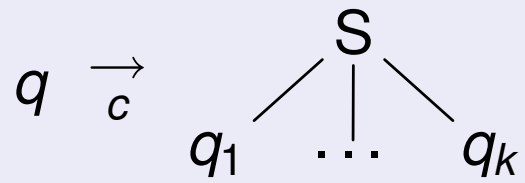
Recall: Weighted Tree Grammar

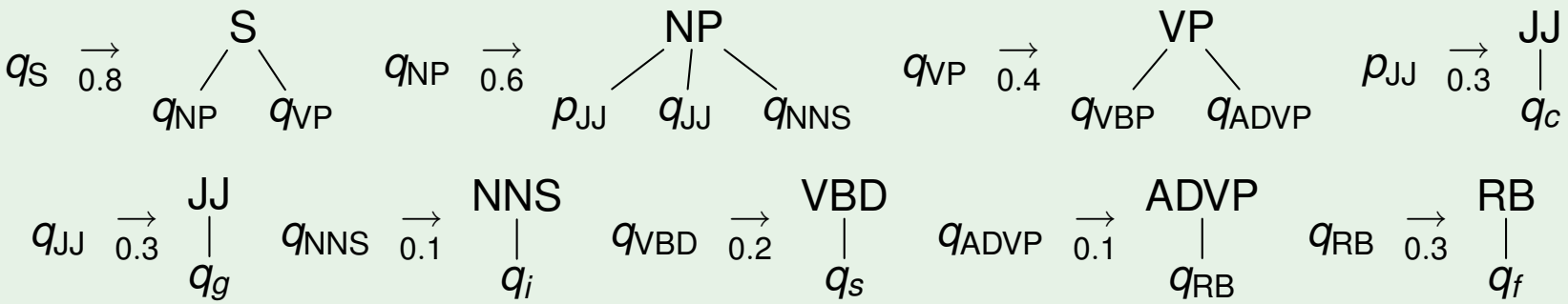
Definition

A regular tree grammar is a grammar with rules of the form



The weights are handled in the same manner as for XTOP (multiply in a derivation & sum different derivation weights)

Example



State Metrics of a Weighted Tree Grammar

Given weighted tree grammar $G = (Q, \Sigma, I, R)$

D_M^q: derivations in M starting from state q

Definition

Inside weight of $q \in Q$:

$$\text{in}(q) = \sum_{d \in D_M^q} \text{wt}(d)$$

Outside weight of $q \in Q$:

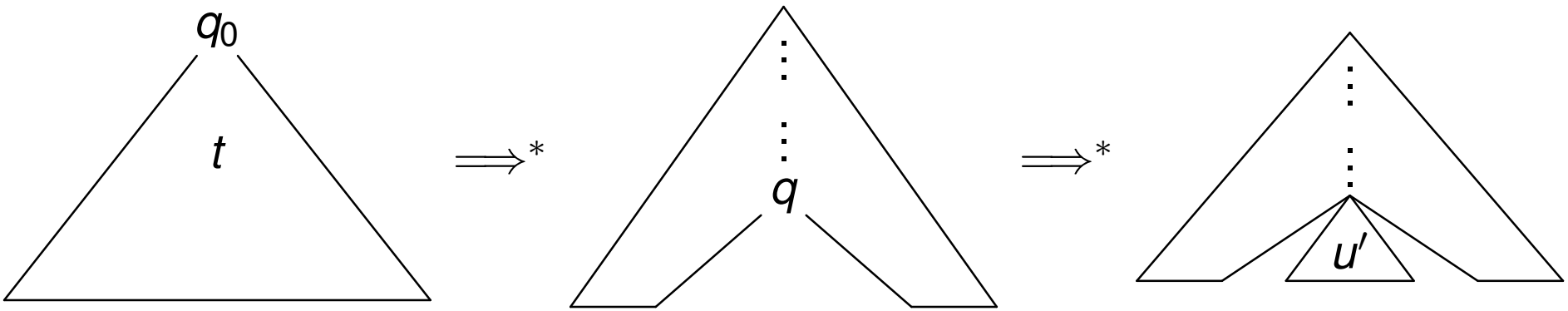
$$\text{out}(q) = \sum_{\substack{t \in C_\Sigma(\{q\}) \\ d \in D_M(t)}} \text{wt}(d)$$

Question

Trivia

Show the relevant part for the

- inside weight of q
- outside weight of q



Expectation Step

Perform the following steps for each input/output pair $(t, u) \in T$ with weight c

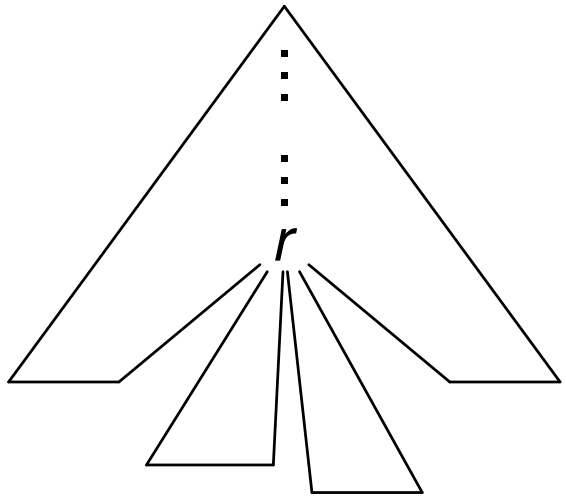
Normalized counts

1. let (Q, R, I, P) be the derivation WTG of M for (t, u)
2. for each production $q \xrightarrow[c']{} r(q_1, \ldots, q_k) \in P$ do

$$\text{count}(r) \leftarrow \text{count}(r) + c \cdot \frac{\text{out}(q) \cdot c' \cdot \prod_{i=1}^{k} \text{in}(q_i)}{M(t, u)}$$

Expectation Step

$$\text{count}(r) \leftarrow \text{count}(r) + c \cdot \frac{\text{out}(q) \cdot c' \cdot \prod_{i=1}^{k} \text{in}(q_i)}{M(t,u)}$$

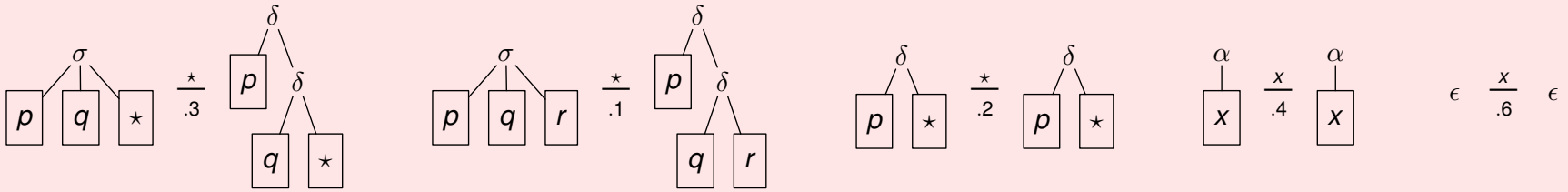


- $\text{out}(q) \cdot c' \cdot \prod_{i=1}^{k} \text{in}(q_i)$: contribution of the rule r
- normalized by $M(t,u)$, the current weight of (t,u)
- this part of the corpus weight c is attributed to r and added to its count

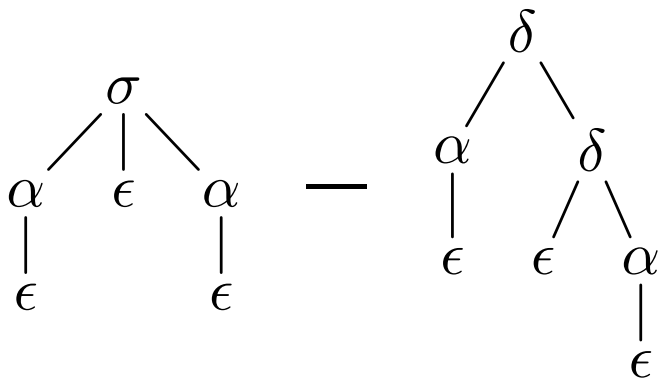
Final Question

Task

Build the derivation WTG for the XTOP with the rules



for the input $\sigma(\alpha(\epsilon), \epsilon, \alpha(\epsilon))$
and the output $\delta(\alpha(\epsilon), \delta(\epsilon, \alpha(\epsilon)))$



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